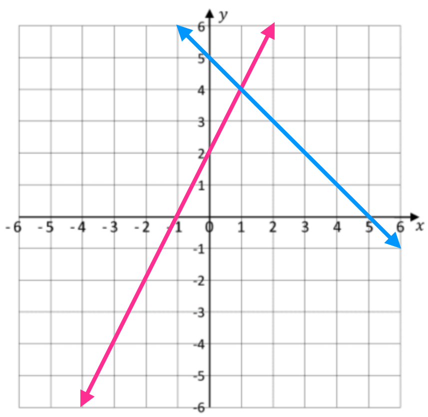
# SOLVING SYSTEMS OF EQUATIONS

***SOLVING A SYSTEM OF EQUATIONS BY GRAPHING***

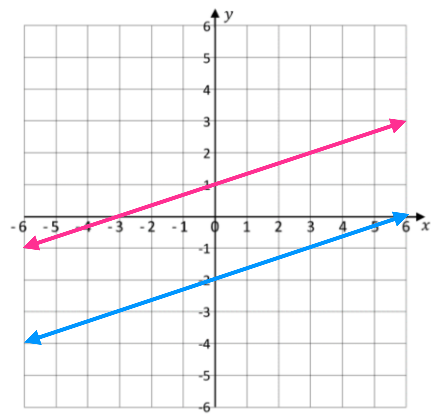



**CASE 1: ONE SOLUTION**
**Graphs intersect at one point.**

The system

$$\boldsymbol{y = 2x + 2}$$
$$\boldsymbol{y = -x + 5}$$

shows two distinct lines that cross exactly at one point. The solution is always written as a (x, y) point.

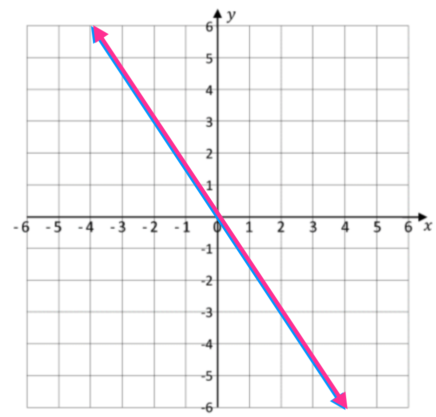



**CASE 2: NO SOLUTION**
**Graphs are parallel.**

The system

$$\boldsymbol{y = \frac{1}{3}x + 1}$$
$$\boldsymbol{y = \frac{1}{3}x - 2}$$

shows two parallel lines that never cross, so there is no intersection. There can be no solution.




**CASE 3: INFINITE SOLUTION**
**Graphs are the same.**

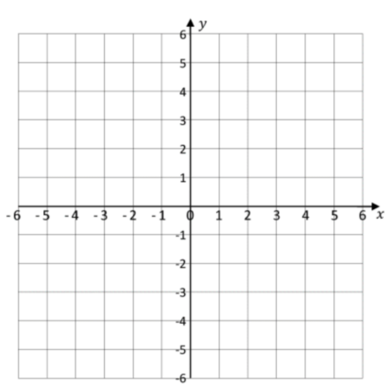
The system

$$\boldsymbol{y = -\frac{3}{2}x}$$
$$\boldsymbol{2y = -3x}$$

appears to show only one line, but it's the same line drawn twice. These two lines intersect at every point along their line. The solution is the whole line, or infinite solutions. It is always written as $\left\{(x, y) | y = -\frac{3}{2}x\right\}$.

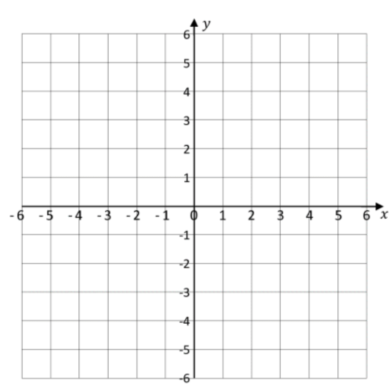
**EXERCISES:** Solve the system graphically.

(1) $y = x - 1$
$y = -x + 5$



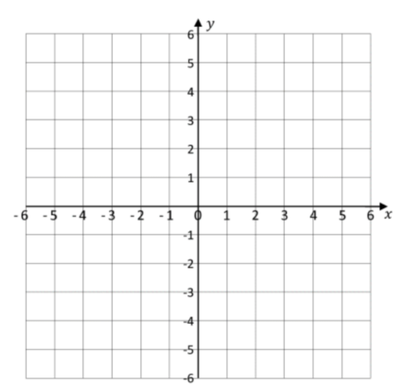

Solution: ____________________

(2) $y = 2x + 3$
$y = 2x + 1$




Solution: ____________________

(3) $y = \frac{1}{4}x + 2$
$4y = x + 8$




Solution: ____________________

## ***SOLVING A SYSTEM OF EQUATIONS BY SUBSTITUTION METHOD***

The ***substitution method*** allows you to substitute one equation into the other equation to eliminate one variable and solve for the remaining variable.

**Example**: Solve the system of equations using substitution method.

$4x + 5y = 10$ ①

$x + 3y = -1$ ②

**Step 1**: Select an equation to solve for one variable. Label this equation as ③.

$x = -1 - 3y$ ③

**Step 2**: Substitute the expression of the isolated variable in Step 1 into the unused original equation to solve for the other variable.

$4(-1 - 3y) + 5y = 10$

$-4 - 12y + 5y = 10$

$-4 - 7y = 10$

$-7y = 14$

$y = -2$

**Step 3**: Substitute the value of the variable in Step 2 into ①,②,or ③ (usually the simpler one) to solve for the other variable.

$x = -1 - 3(-2)$

$x = -1 + 6$

$x = 5$

**Step 4**: Write the final answer as a coordinate. Check.

Solution: $(5, -2)$

$4(5) + 5(-2) = 10$

$20 - 10 = 10$

$10 = 10$ true

$(5) + 3(-2) = -1$

$5 - 6 = -1$

$-1 = -1$ true

**EXERCISES:** Solve the system using substitution method.

(4) $y = 2x - 1$ ①
$4x - 3y = -7$ ②

(5) $2x + 3y = -2$ ①
$x = y + 4$ ②

(6) $5x + 3y = 8$ ①
$-4x + 3y = -1$ ②

(7) $2x + 7y = 1$ ①
$4x + 14y = 3$ ②

(8) $y - 5 = -3x$ ①
$-6x = 2y - 10$ ②

***SOLVING A SYSTEM OF EQUATIONS BY SUBSTITUTION METHOD***

The ***elimination method*** for solving systems of equations involves adding the two equations together.

***Example***: Solve the system using elimination method.

$2x - 3y = 2$ ①
$5x - 7y = 6$ ②

**Step 1:** Choose a variable to eliminate. Multiply one equation by a constant. Repeat for the second equation.

$-5(2x - 3y = 2)$ ①
$2(5x - 7y = 6)$ ②

**Step 2:** Eliminate a variable by adding one equation to the other.

$-10x + 15y = -10$ ①
$\underline{\ \ 10x - 14y = 12}$ ②
$y = 2$

**Step 3:** Substitute the value of the variable into either of the original equations (usually the simpler one) to solve for the other variable.

$5x - 7(2) = 6$
$5x - 14 = 6$
$5x = 20$
$x = 4$

**Step 4**: Write the final answer as a coordinate. Check.

The solution is (4, 2).

| | |
|---|---|
| $-2(4) + 3(2) = -2$ | $5(4) - 7(2) = 6$ |
| $-8 + 6 = -2$ | $20 - 14 = 6$ |
| $-2 = -2$ true | $6 = 6$ true |

***EXERCISE***: Solve the system using elimination method.

(9) $5x - 3y = 13$ ①
$-x + 3y = -5$ ②

(10) $-4x + y = 17$ ①
$4x - 8y = 4$ ②

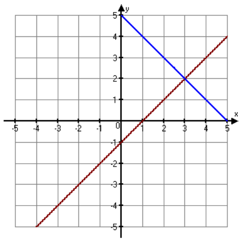
(11) $5x + 2y = 16$ ①
$4x + 3y = 17$ ②

(12) $2x - 3y = 2$ ①
$-4x + 6y = -4$ ②

(13) $8x + 14y = 10$ ①
$4x + 7y = -6$ ②

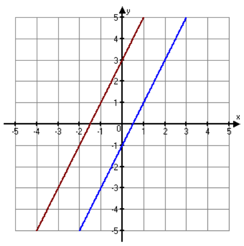
***ANSWERS***

1.) $y = x - 1$
$y = -x + 5$



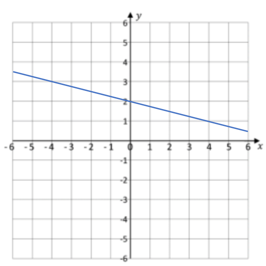
Solution: (3, 2)

2.) $y = 2x + 3$
$y = 2x + 1$



Solution: No solution

3.) $y = \frac{1}{4}x + 2$
$4y = x + 8$



Solution: $\left\{(x, y) \mid y = \frac{1}{4}x + 2\right\}$

4.) (5, 9)
5.) $(2, -2)$
6.) (1, 1)
7.) No solution
8.) $\{(x, y) \mid y - 5 = -3x\}$
9.) $(2, -1)$
10.) $(-5, -3)$
11.) (2, 3)
12.) $\{(x, y) \mid 2x - 3y = 2\}$
13.) No solution